HEAT OUTPUT BY A PARTICULATE SOLID TRANSPORTED BY A GAS IN A PIPE WITH THE HEAT COMPONENT OF THE SOLID VARYING ALONG THE LENGTH

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Analytical relationships are given for the temperature distribution and heat transfer in a compact layer moving in a cylindrical pipe when there are heat sources within the solid whose output varies linearly or exponentially along the length.

A moving compact layer of material may have internal heat production or absorption, as in a chemical reactor, catalyst regenerator, dryer, or other such type of equipment [1-3], so there is considerable interest in heat transfer to the walls of pipes of various shapes. However, virtually all published studies [4-6] relate to immobile beds. Only in [7] do we find a study of heat transfer of a moving bed to walls in a slot for a uniform distribution of the internal heat sources. Here we consider the temperature distribution and heat transfer for a moving bed in a cylindrical pipe when the solid component contains internal heat sources whose output varies along the length.

We use the model of [8]: a bed is considered as a heterogeneous two-component gas—solid system, while each of the components is considered as quasihomogeneous. The conclusions of [8] indicate that such a model reflects satisfactorily the features of heat transfer in such a bed and incorporates the major factors. In formulating and solving the problem we made the following assumptions: 1) the components move in a direct-flow and rod-type fashion; 2) the physical characteristics of the components are constant; 3) the longitudinal heat transport due to thermal conduction is negligible by comparison with the convective transfer; 4) the structure and porosity of the bed do not vary across the cross section. The system of differential equations for the energies of the components and the heat transfer at the boundary takes the following form:

$$\rho_g c_{pg}(1-\beta) v_{g0} \frac{\partial \vartheta_g}{\partial x} = \lambda_g^* \left(\frac{\partial^2 \vartheta_g}{\partial r^2} + \frac{1}{r} \frac{\partial \vartheta_g}{\partial r} \right) - \alpha_m a (\vartheta_s - \vartheta_g); \quad (1)$$

$$\rho_s c_s \beta v_{s0} \frac{\partial \vartheta_s}{\partial x} = \lambda_s^* \left(\frac{\partial^2 \vartheta_s}{\partial r^2} + \frac{1}{r} \frac{\partial \vartheta_s}{\partial r} \right) + \alpha_m a (\vartheta_s - \vartheta_g) \pm q_{vs}(x); \quad (2)$$

$$\alpha = -\left[\lambda_g^* \left(\frac{\partial \vartheta_g}{\partial r} \right)_{r=r_0} + \lambda_s^* \left(\frac{\partial \vartheta_s}{\partial r} \right)_{r=r_0} \right] \bar{\vartheta}_f^{-1}; \quad (3)$$

$$x \geqslant 0,\ 0 \leqslant r \leqslant r_0:\ v_g = v_{g0} = \text{const};\ v_s = v_{s0} = \text{const}; \quad (4a)$$

$$x = 0,\ 0 \leqslant r \leqslant r_0:\ \vartheta_g = \vartheta_{g0} = \text{const};\ \vartheta_s = \vartheta_{s0} = \text{const}; \quad (4b)$$

$$x > 0,\ r = r_0:\ \vartheta_g = \vartheta_s = 0; \quad (4c)$$

$$x \geqslant 0,\ r = 0:\ \frac{\partial \vartheta_g}{\partial r} = \frac{\partial \vartheta_s}{\partial r} = 0; \quad (4d)$$

$$x > 0;\ t_w = \text{const};\ q_{vs} = f(x). \quad (4e)$$

In the boundary conditions of the first kind in (4c) it has been assumed that the temperatures of the components at the boundary are equal to the surface temperature, i.e., the thermal resistance at the wall is negligible. The analysis of [9] indicates that this assumption

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is approximately correct for unhindered motion of the bed, provided that the contact time is sufficient and the roughness is slight. Under these conditions, the analytical relationships derived from this model agree with experiment within the error of the latter.

We consider two cases of variation in the output of capacity of the heat sources or sinks, namely, varying linearly and exponentially along the length:

$$q_{vs} = q_{vs0}(1 + bx); \tag{5a}$$

$$q_{vs} = q_{vs0}\, e^{mx}. \tag{5b}$$

The solution is derived as in [8] by double integral transform (Laplace transform with respect to the x coordinate and Hankel transform with respect to the r coordinate).

If the output of the internal sources varies linearly, we get the following equations for the temperatures of the components and the local heat-transfer coefficient:

$$\theta_g = \frac{\vartheta_g}{\vartheta_{g0}} = \sum_{n=1}^{\infty} \frac{2J_0(\mu_n R)}{\mu_n J_1(\mu_n)} \left[(Q_{1n} + Q'_{1n}) \exp(p_{1n}x) - (Q_{2n} + Q'_{2n}) \exp(p_{2n}x) + \right.$$
$$\left. + \frac{H_s B_1 (1 + bx)}{p_{1n} p_{2n} \vartheta_{g0}} + \frac{b H_s B_1 (p_{1n} + p_{2n})}{p_{1n}^2 p_{2n}^2 \vartheta_{g0}} \right]; \tag{6}$$

$$\theta_s = \frac{\vartheta_s}{\vartheta_{g0}} = \sum_{n=1}^{\infty} \frac{2B_2 J_0(\mu_n R)}{\mu_n J_1(\mu_n)} \left\{ \left(\frac{Q_{1n}}{p_{1n} + D_n} + Q''_{1n} \right) \exp(p_{1n}x) - \left(\frac{Q_{2n}}{p_{2n} + D_n} + Q''_{2n} \right) \exp(p_{2n}x) - \right.$$
$$\left. - \frac{(p_{1n} + p_{2n} + D_n) H_s (1 + bx)}{p_{1n} p_{2n} \vartheta_{g0} B_2} + \frac{b H_s [p_{1n} p_{2n} - (p_{1n} + p_{2n})(p_{1n} + p_{2n} + D_n)]}{p_{1n}^2 p_{2n}^2 \vartheta_{g0} B_2} \right\}; \tag{7}$$

$$\alpha = \frac{W_g + W_s}{2r_0} \left\{ \lambda_g^* \sum_{n=1}^{\infty} \left[(Q_{1n} - Q'_{1n}) \exp(p_{1n}x) - (Q_{2n} + Q'_{2n}) \exp(p_{2n}x) + \frac{H_s B_1 (1 - bx)}{p_{1n} p_{2n} \vartheta_{g0}} + \frac{b H_s B_1 (p_{1n} + p_{2n})}{p_{1n}^2 p_{2n}^2 \vartheta_{g0}} \right] + \right.$$
$$+ \lambda_s^* B_2 \sum_{n=1}^{\infty} \left[\left(\frac{Q_{1n}}{p_{1n} + D_n} + Q''_{1n} \right) \exp(p_{1n}x) - \left(\frac{Q_{2n}}{p_{2n} + D_n} + Q''_{2n} \right) \times \right.$$
$$\left.\left. \times \exp(p_{2n}x) - \frac{(p_{1n} + p_{2n} + D_n) H_s (1 + bx)}{p_{1n} p_{2n} \vartheta_{g0} B_2} + \frac{b H_s [p_{1n} p_{2n} - (p_{1n} + p_{2n})(p_{1n} + p_{2n} + D_n)]}{p_{1n}^2 p_{2n}^2 \vartheta_{g0} B_2} \right] \right\} \times$$
$$\times \left\{ \sum_{n=1}^{\infty} \frac{W_g}{\mu_n^2} \left[(Q_{1n} + Q'_{1n}) \exp(p_{1n}x) - (Q_{2n} + Q'_{2n}) \exp(p_{2n}x) + \frac{H_s B_1 (1 + bx)}{p_{1n} p_{2n} \vartheta_{g0}} + \frac{b H_s B_1 (p_{1n} + p_{2n})}{p_{1n}^2 p_{2n}^2 \vartheta_{g0}} \right] + \right.$$
$$+ \sum_{n=1}^{\infty} \frac{B_2 W_s}{\mu_n^2} \left[\left(\frac{Q_{1n}}{p_{1n} + D_n} + Q''_{1n} \right) \exp(p_{1n}x) - \left(\frac{Q_{2n}}{p_{2n} + D_n} + Q''_{2n} \right) \exp(p_{2n}x) - \frac{(p_{1n} + p_{2n} + D_n) H_s (1 + bx)}{p_{1n} p_{2n} \vartheta_{g0} B_2} + \right.$$
$$\left.\left. + \frac{b H_s [p_{1n} p_{2n} - (p_{1n} + p_{2n})(p_{1n} + p_{2n} + D_n)]}{p_{1n}^2 p_{2n}^2 \vartheta_{g0} B_2} \right] \right\}^{-1}. \tag{8}$$

In (6)-(8) the summation is carried over all positive roots of the characteristic equation $J_0(\mu_n) = 0$; here

$$\mu_n = k_n r_0; \quad R = \frac{r}{r_0}; \quad p_{1n} = \frac{-E_n + \sqrt{E_n^2 - 4N_n}}{2};$$
$$p_{2n} = \frac{-E_n - \sqrt{E_n^2 - 4N_n}}{2};$$
$$E_n = A_1 k_n^2 + A_2 k_n^2 + B_1 + B_2;$$
$$N_n = A_1 A_2 k_n^4 + A_1 k_n^2 B_2 + A_2 k_n^2 B_1;$$
$$M_n = A_2 k_n^2 + \frac{\vartheta_{s0}}{\vartheta_{s0}} B_1 + B_2; \quad D_n = A_2 k_n^2 + B_2; \tag{9}$$
$$A_1 = \frac{\lambda_g^*}{\rho_g c_{pg} (1 - \beta) v_{g0}}; \quad A_2 = \frac{\lambda_s^*}{\rho_s c_s \beta v_{s0}}; \quad a = \frac{6\beta}{d};$$

$$B_1 = \frac{\alpha_m a}{\rho_g c_{pg}(1-\beta) v_{g0}}; \quad B_2 = \frac{\lambda_s^*}{\rho_s c_s \beta v_{s0}}; \quad H_s = \frac{q_{vs0}}{\rho_s c_s \beta v_{s0}};$$

$$Q_{1n} = \frac{p_{1n} + M_n + \dfrac{B_1 H_s}{\vartheta_{g0} p_{1n}}}{p_{1n} - p_{2n}}; \quad Q_{2n} = \frac{p_{2n} + M_n + \dfrac{B_1 H_s}{\vartheta_{g0} p_{2n}}}{p_{1n} - p_{2n}};$$

$$Q'_{1n} = \frac{bH_s B_1}{p_{1n}^2 (p_{1n} - p_{2n}) \vartheta_{g0}}; \quad Q'_{2n} = \frac{bH_s B_1}{p_{2n}^2 (p_{1n} - p_{2n}) \vartheta_{g0}}; \tag{10}$$

$$Q''_{1n} = -\frac{b(p_{2n} + D_n) H_s}{p_{1n}^2 (p_{1n} - p_{2n}) B_2 \vartheta_{g0}}; \quad Q''_{2n} = -\frac{b(p_{1n} + D_n) H_s}{p_{2n}^2 (p_{1n} - p_{2n}) B_2 \vartheta_{g0}}.$$

If the variation is exponential, the temperature and heat-transfer rate distributions are described by

$$\theta_g = \sum_{n=1}^{\infty} \frac{2J_0(\mu_n R)}{\mu_n J_1(\mu_n)} \left[Q_{1n} \exp(p_{1n}x) - Q_{2n} \exp(p_{2n}x) + \frac{B_1 H_s}{(m - p_{1n})(m - p_{2n}) \vartheta_{g0}} \exp(mx) \right]; \tag{11}$$

$$\theta_s = \sum_{n=1}^{\infty} \frac{2B_2 J_0(\mu_n R)}{\mu_n J_1(\mu_n)} \left[\frac{Q_{1n}}{p_{1n} + D_n} \exp(p_{1n}x) - \frac{Q_{2n}}{p_{2n} + D_n} \times \right.$$
$$\left. \times \exp(p_{2n}x) + \frac{H_s(m - p_{1n} - p_{2n} - D_n)}{(m - p_{1n})(m - p_{2n}) B_2 \vartheta_{g0}} \exp(mx) \right]; \tag{12}$$

$$\alpha = \frac{W_g + W_s}{2r_0} \left\{ \lambda_g^* \sum_{n=1}^{\infty} \left[Q_{1n} \exp(p_{1n}x) - Q_{2n} \exp(p_{2n}x) + \frac{B_1 H_s \exp(mx)}{(m - p_{1n})(m - p_{2n}) \vartheta_{g0}} \right] + \right.$$
$$\left. + \lambda_s^* B_2 \sum_{n=1}^{\infty} \left[\frac{Q_{1n}}{p_{1n} + D_n} \exp(p_{1n}x) - \frac{Q_{2n}}{p_{2n} + D_n} \exp(p_{2n}x) + \frac{H_s(m - p_{1n} - p_{2n} - D_n)}{(m - p_{1n})(m - p_{2n}) B_2 \vartheta_{g0}} \exp(mx) \right] \right\} \times$$
$$\times \left\{ \sum_{n=1}^{\infty} \frac{W_g}{\mu_n^2} \left[Q_{1n} \exp(p_{1n}x) - Q_{2n} \exp(p_{2n}x) + \frac{B_1 H_s \exp(mx)}{(m - p_{1n})(m - p_{2n}) \vartheta_{g0}} \right] + \right.$$
$$\left. + \sum_{n=1}^{\infty} \frac{W_s B_2}{\mu_n^2} \left[\frac{Q_{1n}}{p_{1n} + D_n} \exp(p_{1n}x) - \frac{Q_{2n}}{p_{2n} + D_n} \exp(p_{2n}x) + \frac{H_s(m - p_{1n} - p_{2n} - D_n)}{(m - p_{1n})(m - p_{2n}) B_2 \vartheta_{g0}} \exp(mx) \right] \right\}^{-1}. \tag{13}$$

Here

$$Q_{1n} = \frac{p_{1n} + M_n + \dfrac{H_s B_1}{(p_{1n} - m) \vartheta_{g0}}}{p_{1n} - p_{2n}};$$

$$Q_{2n} = \frac{p_{2n} + M_n + \dfrac{H_s B_1}{(p_{2n} - m) \vartheta_{g0}}}{p_{1n} - p_{2n}}. \tag{14}$$

These equations allow one to perform a quantitative analysis of the effects of the major parameters on the heat transfer and temperature distribution: the velocities and physical parameters of the components, the geometrical characteristics of the pipe and particles, the output from the heat sources, and the mode of variation along the length. The formulas are applicable to various cases: heat production ($q_{vs} > 0$) and heat absorption ($q_{vs} < 0$) in the solid component, heat supply via the wall ($\vartheta_{f0} = t_{f0} - t_{ws} < 0$) and heat removal via the wall ($\vartheta_{f0} > 0$), as well as a fall in output along the length ($b < 0$, $m < 0$) and an increase in output in that direction ($b > 0$, $m > 0$).

Calculations show that in (6)-(8) and (11)-(13) one can neglect terms containing $\exp(p_{2n}x)$ by comparison with terms containing $\exp(p_{1n}x)$, since $|p_{1n}| \ll |p_{2n}|$; in the linear case, the terms containing $\exp(p_{1n}x)$ in (6)-(8) become negligible at a sufficient distance from the inlet, so the temperature distribution is determined mainly by the output from the heat sources and the law followed by the latter. The last factor is responsible for the variation in the heat-transfer coefficient along the flow. The absolute output from the sources has no effect on the heat transfer in this region. If the source output declines exponentially along the length, then (11)-(13) constitute convergent series. One need take only the first term in each of these formulas at sufficiently large values of the longitudinal coordinate, and then the radial temperature distribution remains constant from point to point. The component temperatures and the heat-transfer coefficient are determined by the source output, as are the variations along the flow.

If b = 0 and m = 0, (6)-(8) and (11)-(13) describe the temperature distribution and heat transfer for a moving bed with heat sources uniformly distributed in the solid component. If q_{vs} = const, the excess temperatures of the components and the general mode of variation along the flow (decrease or increase) are determined by the source output. The heat-transfer coefficient decreases along the channel, while any increase in the source output results in some acceleration of the heat transfer. At some distance from the inlet dependent on the working conditions and geometrical characteristics, the solution becomes of self-modeling type in that the excess temperatures of the components and the heat-transfer coefficients cease to vary along the channel. The heat-transfer coefficient is independent of the source output in this region. These conclusions agree with results for slot channels [7].

If $q_{vs} = 0$, (6)-(13) become the relationships of [8] for a moving layer without heat sources.

These results can be used in calculations on the temperature distribution and heat transfer to the wall for cylindrical systems with dense beds that absorb or produce heat.

NOTATION

a, particle surface per unit volume; c, specific heat; d, particle size; q_{vs}, capacity of internal heat sources; r, R, current radii; r_0, channel radius; v, velocity; W, water equivalent; β, bulk concentration of solid component; λ^*, effective thermal conductivity of component in the bed; $\vartheta = t - t_{ws}$; $\theta = \vartheta / \vartheta_{go}$, excess temperature. Indices: g, gas component; f, flow; w, wall; s, solid component; 0, inlet cross section (x = 0).

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